$$\omega_{cr} = (Z_{ef})_{cr} + T_{cr}\left[\frac{\partial (Z_{ef})_{cr}}{\partial T}\right]_{p_{cr}}. \qquad (11')$$

In conclusion, we would like to draw attention to the fact that Eqs. (17) and (31) are original because they combine thermal quantities and the flow velocity. On the other hand, the invariance makes it possible to solve further problems of applied thermogasdynamics and to investigate a complicated process of flow (dissociation of molecules, etc.). Studies of the obtained dependences disclose fundamental features of the motion of an arbitrary gas and promote the solution of problems which are of actual significance in contemporary technology involving high-speed flows of real gases.

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SOME FEATURES OF THE THERMAL STRATIFICATION OF LIQUIDS DURING NATURAL CONVECTION IN CYLINDRICAL CAVITIES WITH ANNULAR RIBS

V. A. Evstaf'ev

UDC 536.252:621.642.03

The effect of annular ribs on the temperature of the free surface of a liquid during natural convection in a heated vertical cylindrical cavity has been investigated. A method is proposed for calculating the limiting value of this temperature.

Free convective motions occur in a liquid during heat exchange between a container or cavity and the surroundings which lead to nonlinear temperature distributions over its volume, or thermal stratification. This thermal stratification can have a significant effect on the intensity of the thermophysical processes occurring in the container, as a result of which its study is of practical interest.

Real containers often contain framing elements or other devices which are positioned in the zone of the free-convection boundary layer and which have an effect on the temperature distribution over the volume of the liquid. An experimental investigation into the effect of ribs on thermal stratification was carried out in [1-3]. It was found that the presence of ribs leads to a decrease in the vertical temperature gradient of the liquid on average with respect to the height of the part of the container. The results of experiments with visualization of the liquid flows have been given. It was noted in [1] that the temperature of the free surface of the liquid in the container with ribs was somewhat higher than in the container without ribs, while the opposite result was found in [2, 3]. There are no methods available for calculating the thermal stratification in containers with ribs.

In the present paper an investigation has been made into the effect of annular ribs on the maximum temperature of a liquid in a heated cavity or container, which is the temperature of the free surface, and a method is proposed for calculating the maximum value of this temperature. For carrying out the experiments two cylindrical containers were used (Fig. 1) with heights of 0.25 and 0.75 m and diameters of 0.25 m. The cylindrical part of each vessel was made of stainless steel of thickness $2 \cdot 10^{-3}$ m and at the bottom end had a

D. F. Ustinov Institute of Mechanics, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 5, pp. 759-763, November, 1988. Original article submitted July 7, 1987.

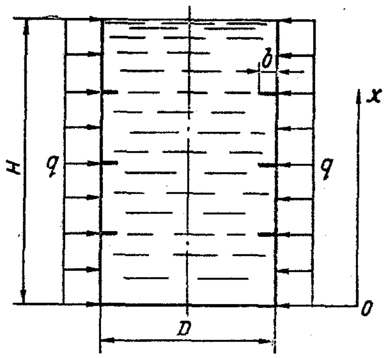

Fig. 1

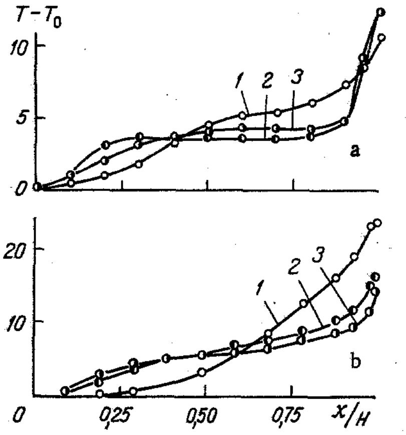

Fig. 2

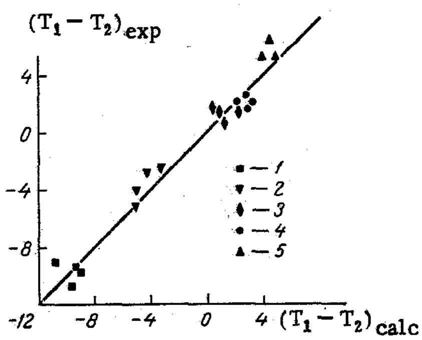

Fig. 3

Fig. 1. Sketch of vessel with annular ribs.

Fig. 2. Distributions of the water temperature over the height of the container: a) $H = 0.75$ m, $q = 6 \cdot 10^3$ W/m², $T_0 = 292$ K, $\tau = 240$ sec, 1) $n = 0$; 2) $n = 3$, $b = 2.5 \cdot 10^{-2}$ m; 3) $n = 3$, $b = 4 \cdot 10^{-2}$ m; b) $H = 0.25$ m, $q = 1.2 \cdot 10^4$ W/m², $T_0 = 296$ K, $\tau = 180$ sec, 1) $n = 0$; 2) $n = 6$, $b = 10^{-2}$ m; 3) $n = 6$, $b = 4 \cdot 10^{-2}$ m. $T - T_0$ is given in K.

Fig. 3. Experimental and calculated values of the temperature difference $T_1 - T_2$ for water: 1) $H = 0.25$ m, $n = 6$, $q = 1.2 \cdot 10^4$ W/m², $T_0 = 296$ K, $\tau = 120$-300 sec; 2) $H = 0.25$ m, $n = 2$, $q = 1.2 \cdot 10^4$ W/m², $T_0 = 296$ K, $\tau = 120$-300 sec; 3) $H = 0.75$ m, $n = 9$, $q = 6 \cdot 10^3$ W/m², $T_0 = 292$ K, $\tau = 120$-300 sec; 4) $H = 0.75$ m, $n = 3$, $q = 6 \cdot 10^3$ W/m², $T_0 = 292$ K, $\tau = 120$-300 sec; 5) $H = 0.75$ m, $n = 3$, $q = 8.3 \cdot 10^3$ W/m², $T_0 = 297$ K, $\tau = 120$-180 sec. $T_1 - T_2$ in K.

flange for connecting it to a flat textolite base of thickness $1.2 \cdot 10^{-2}$ m. The liquid (water) was heated by means of electric heaters placed on the outside surface of the cylinder. The ribs were placed so that they divided the height of the vessel into n + 1 equal parts. The liquid temperature was measured by thermocouples placed along the axis of symmetry of the container. The temperature changes in the radial direction outside of the boundary layer did not exceed the experimental error. The error in determining the temperature was ±0.6 K, while the error in determining a temperature difference measured by a given thermocouple was ±0.2 K. The experimental parameters were varied over the following ranges: $Ra^* = 1.1 \cdot 10^{11}$-$2.2 \cdot 10^{14}$; $Pr = 4.7$-7.5; $Fo = 1.5 \cdot 10^{-5}$-$1.5 \cdot 10^{-3}$; $n = 0$-9. The ribs had widths of $5 \cdot 10^{-3}$, 10^{-2}, $2.5 \cdot 10^{-2}$, and $4 \cdot 10^{-2}$ m. The values indicated above for the Ra*, Pr, and Fo numbers correspond to the initial temperature of the liquid.

The experiments which have been carried out showed that the effect of the ribs on the temperature of the free liquid surface is not unique, in contrast to their effect on the liquid temperatures in the middle part of the container. The temperature of the liquid at the free surface in the containers with ribs may be either lower or higher than the analogous temperatures in the case of smooth containers (Fig. 2). As the rib width increased the temperature of the free surface varied (increased or decreased) and then remained approximately constant. The initial change in temperature of the free surface can be explained by a transition from an uninterrupted flow in the free-convection boundary layer to an interrupted flow. The transition process terminates at some value of the rib width: an independent boundary layer is formed above the ribs, which is confirmed by the data of [1-3] and by the results of our experiments with visualization of the flow. The temperature of the free surface of the liquid with a uniform spacing of the ribs is mainly determined by the boundary layer parameters above the upper rib. As the rib width is increased still further these parameters essentially cease to vary, which explains the constancy of the free surface temperature which then results. However, in our opinion some change of the liquid temperature in the middle part of the container can still occur, since the rib width influences the conditions of mixing of the free-convection streams passing round the ribs.

It should be noted that the flow picture of the liquid in the volume between the liquid surface and the upper annular rib has a great deal in common with the flow picture in a smooth container of the same height with side and bottom heating for large rib widths, when the limiting value of the free surface temperature occurs. In both cases there is an ascending free-convection stream of liquid in the lower part of the volume: in the first case the stream starts from under the rib, and in the second case from the bottom of the container. According to [4], bottom heating does not have an effect on the free surface temperature until the temperature of the liquid is no longer uniform over the whole volume.

The analogy which has been considered makes it possible to assume that with side heating the limiting temperature of the free liquid surface in a container of height H with annular ribs is equal to the temperature of the free surface in a container with smooth walls of height $h = H/(n + 1)$. In order to check the acceptability of this assumption a comparison was made of the experimental and calculated data on the limiting temperatures of the free liquid surfaces in containers with ribs T_1. The results of the calculations are shown in Fig. 3 in the form of the difference in temperature $T_1 - T_2$. As the experimental values T_1 use was made of the values of the liquid surface temperature for $b = 4 \cdot 10^{-2}$ m. T_1 and T_2 were calculated from the relationships in [4] which describe the changes in the temperatures of the free liquid surfaces during laminar and turbulent free convection in smooth containers heated through the cylindrical surfaces (q = const) up to the moment when the zone of stratification of the bottom was attained. The relationship for laminar convection was obtained by using an approximation to the numerical solution of the equation of the laminar free-convection boundary layer about a vertical plate (Pr = 1-30), and that for turbulent convection by using an analytical solution of the integral equation of the turbulent boundary layer. As can be seen from the figure, the agreement between the experimental and calculated data is satisfactory.

By replacing h by $H/(n + 1)$ in these relationships the following expression can be obtained connecting the temperature of the free liquid surface in the container to the number of ribs:

for laminar free convection:

$$\frac{(T_1 - T_0)\lambda}{qH} = \frac{AI^{-1}}{1 - [1 - B(n+1)^{\frac{1}{5}}]^5}, \tag{1}$$

and for turbulent free convection:

$$\frac{(T_1 - T_0)\lambda}{qH} = \frac{AI^{-1}}{1 - [1 + C(n+1)^{-\frac{1}{7}}]^{-7}}, \tag{2}$$

where

$$A = 4\xi\,\mathrm{Fo};\quad B = 1.26\xi\,\mathrm{Fo}\,\mathrm{Pr}^{0,612}\,\mathrm{Gr}^{*\frac{1}{5}};$$

$$C = 0.185\xi\,\mathrm{Fo}(1 + 0.443\,\mathrm{Pr}^{\frac{2}{3}})\,\mathrm{Pr}^{\frac{1}{3}}\,\mathrm{Gr}^{*\frac{2}{7}};$$

and I is the energy integral, which depends only on $Gr^*/(n + 1)^4$. Its values have been obtained experimentally for $Gr^*/(n + 1)^4 = 10^8$-$2 \cdot 10^{15}$, Pr = 1.3-32, ξ = 1.15 and 2.35 [4, 5]. For laminar free convection ($Ra^*/(n + 1)^4 \leq 10^{11}$) I is an increasing function of n, while for turbulent free convection it is a decreasing function.

Equation (1) can be applied until the denominator of the right-hand side is longer equal to unity, which corresponds to the moment of reaching the zone of thermal stratification of the bottom of the smooth container of height h. For relationship (2) it is recommended to take the corresponding limitation as being equal to 0.95 [4], since the denominator of the right-hand side approaches unity asymptotically.

An analysis of relationships (1) and (2) shows that during laminar free convection in the container the temperature of the free liquid surface decreases as the number of ribs increases, while in turbulent free convection it increases, and hence during laminar free convection the temperature of the free liquid surface in a container with ribs is lower than in a smooth container, while during turbulent free convection the opposite is the case.

In our experiments the temperature of the free liquid surface practically reached the limiting value for values of the ratio of the width of the rib to the thickness of the boundary layer below it equal to 2-4. The boundary layer thickness was determined approximately from the results of experiments with visualization of the liquid flows, which were carried out on two models having the shapes of vertical parallelepipeds under conditions close to those which occurred in the cylindrical containers. The internal volumes of the models were 0.186 × 0.12 × 0.06 m and 0.7 × 0.18 × 0.09 m. Each model had two transparent walls of plexiglass, two steel walls with ribs, and a textolite bottom. Electric heaters were placed on the outside surfaces of the steel walls. The models were filled with an orange-colored solution consisting of water (<99.98% by weight), hydrochloric acid, caustic soda, and thymol blue indicator. An electrical current of voltage 1.5-2 V was passed through the solution as a result of which the liquid close to the cathode took on a violet color due to the change in acidity. Copper wires of diameter 0.1 mm stretched between the steel walls were used as the cathodes, while a copper rod of diameter 1.5 mm fixed along the vertical axis of the model served as the anode. The violet-colored liquid was entrained in the free-convection motion, making it possible to carry out visual observations of the latter.

NOTATION

H, height of the liquid column in the container; D, diameter of the container; n, number of ribs; b, width of rib; h, distance between ribs; x, coordinate; q, heat flux density; T, liquid temperature; T_0, initial liquid temperature; T_1, limiting temperature of the free liquid surface in container with ribs; T_2, temperature of the free surface of the liquid in a container with smooth walls of height H; τ, time elapsed from the start of the process of thermal stratification of the liquid in the container up to the moment being considered; $\xi = H/D$, relative height of the liquid column; $Gr^* = g\beta qH^4/\lambda\nu^2$, modified Grashof number; Pr, Prandtl number; $Ra^* = Gr^*Pr$, modified Rayleigh number; $Fo = a\tau/H^2$, Fourier number; g, accelerating force of gravity; β, λ, a, ν, coefficients of volumetric expansion, thermal conduction, temperature conduction, and kinematic viscosity of the liquid. Subscripts: exp, experiment; calc, calculated.

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